Math 251 Midterm 2 Sample

Name: _____

This exam has 8 questions, for a total of 100 points.

Please answer each question in the space provided. You need to write **full solutions**. Answers without justification will not be graded. Cross out anything the grader should ignore and circle or box the final answer.

Question	Points	Score
1	10	
2	14	
3	14	
4	10	
5	10	
6	10	
7	12	
8	20	
Total:	100	

Question 1. (10 pts)

$$f(x, y, z) = z\sqrt{y^2 - x}$$

(a) Find the gradient of the function $f(x, y, z)$

(b) Find the maximum rate of change of $f(x, y, z)$ at the point $(5, 3, 1)$.

Question 2. (14 pts)

Given

$$f(x, y) = x^2y - x^2 - y^2$$

Determine all local maximum, minimum and saddle points.

Question 3. (14 pts)

Use the Lagrange multiplier method to find the absolute extreme values of the function

$$f(x, y) = xy$$

with the constraint $x^2 + 4y^2 = 8$.

Question 4. (10 pts)

Rewrite (but do not evaluate)

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x+y)dydx$$

in polar coordinates.

Question 5. (10 pts)

Rewrite (but do not evaluate)

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}}(x+z)\,dzdxdy$$

in spherical coordinates.

Question 6. (10 pts)

Evaluate the following integral by switching the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} \, dxdy$$

Question 7. (12 pts)

Find the area of the region above the circle $x^2 + y^2 = 4y$ and below the circle $x^2 + y^2 = 4$.

Question 8. (20 pts)

E is the solid that is between the upper half of the sphere $x^2+y^2+z^2=4$ and the cone $z=\sqrt{x^2+y^2}$.

(a) Write the volume of E as a triple integral in xyz-coordinates.

(b) Write the volume of E as a triple integral in cylindrical coordinates.

(c) Write the volume of E as a triple integral in spherical coordinates.

(d) Use one of your answers from part (a), (b) and (c) to calculate the volume of E.